Massive stars

School on “The synthesis of the elements”

Granada (12-16 April 2010)

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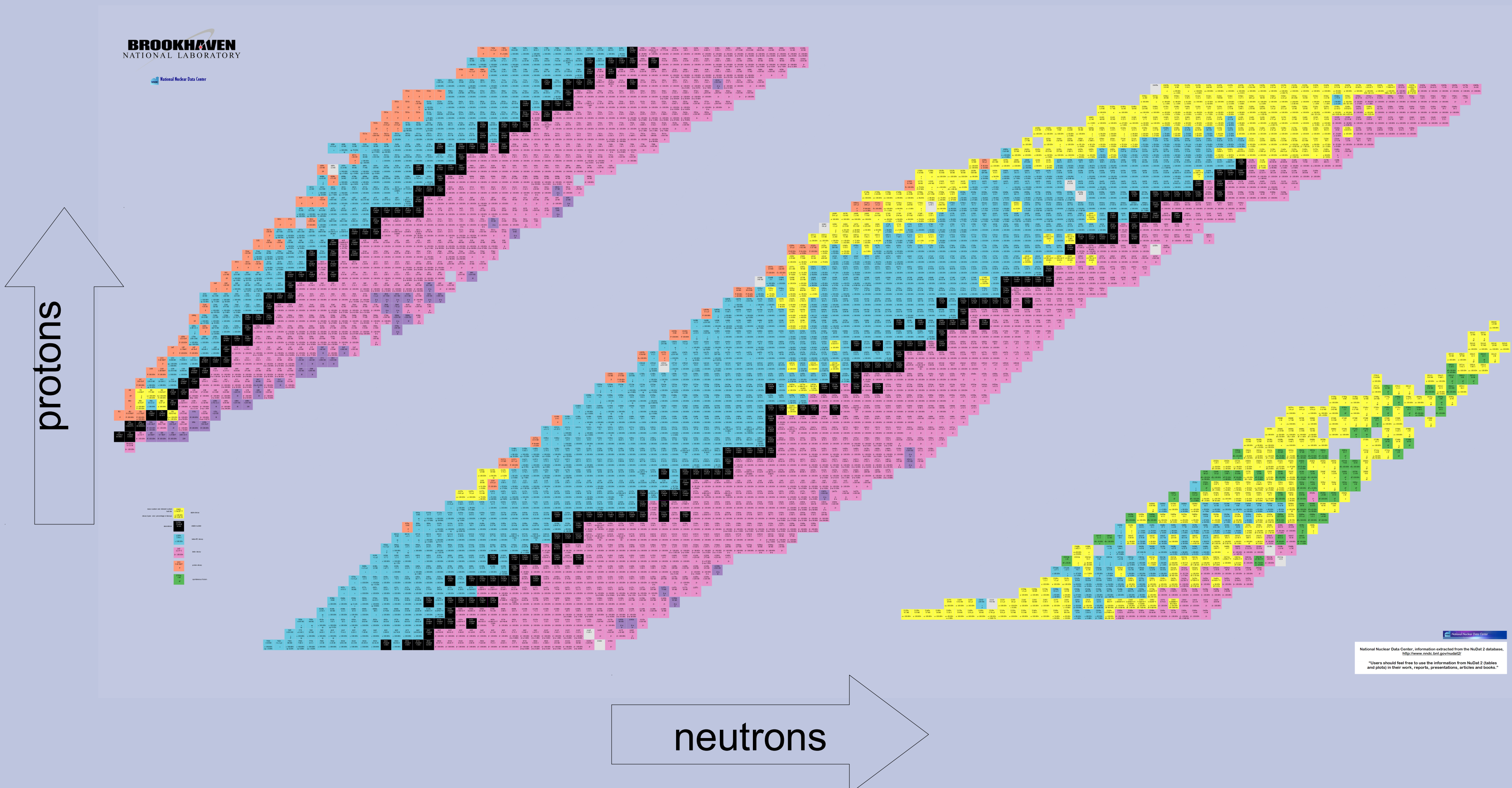
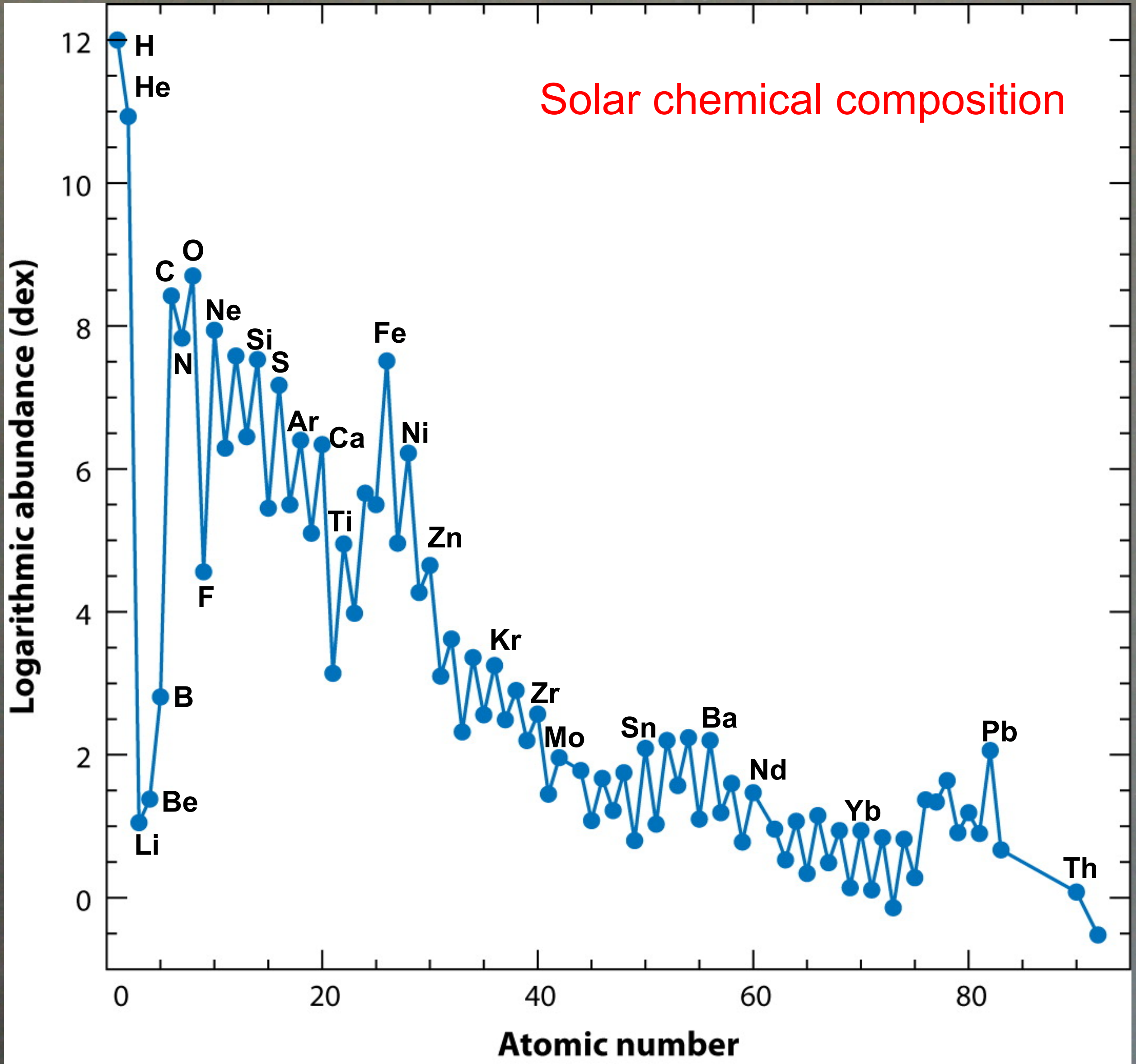
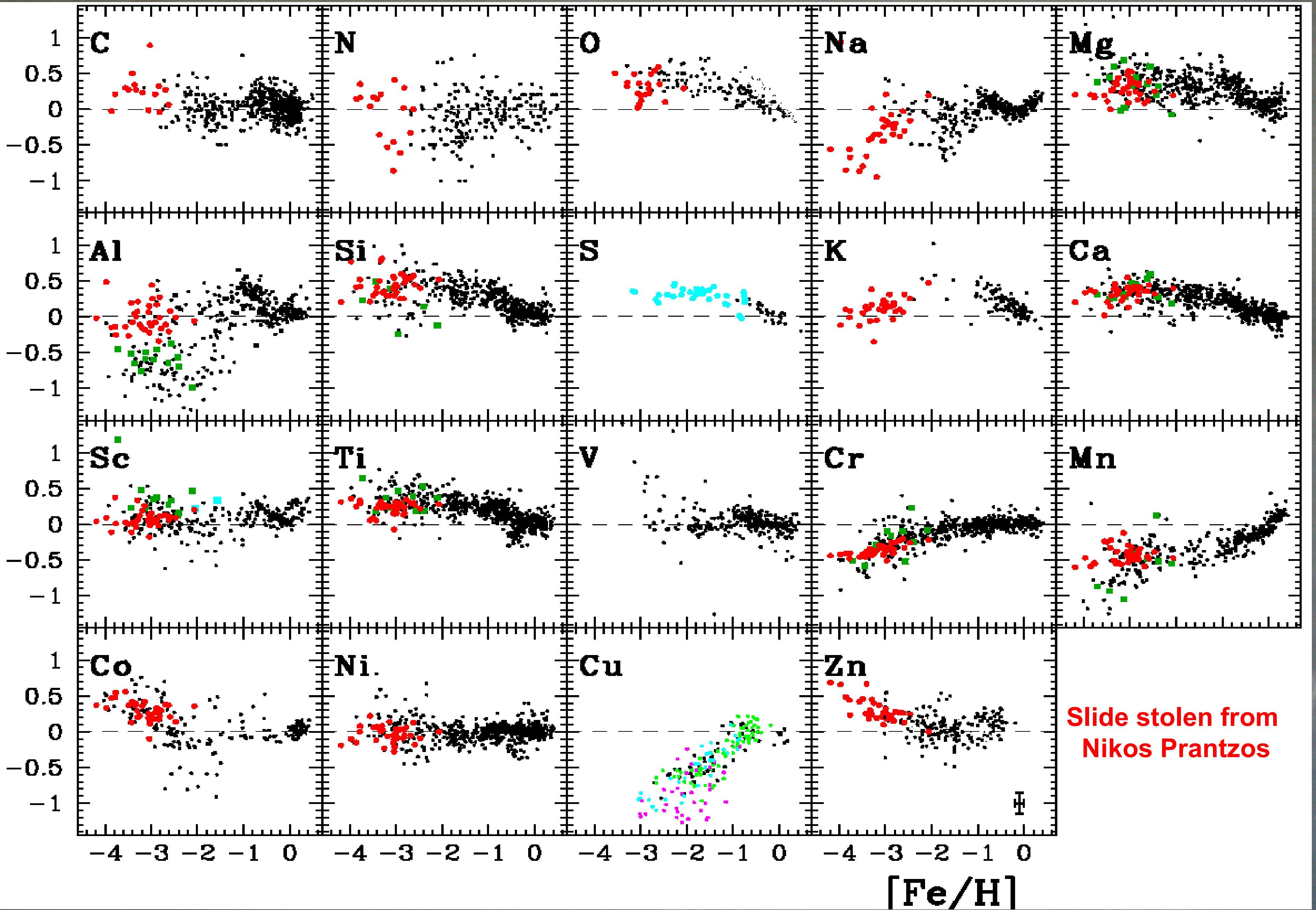
Part 1

TABLE OF NUCLIDES

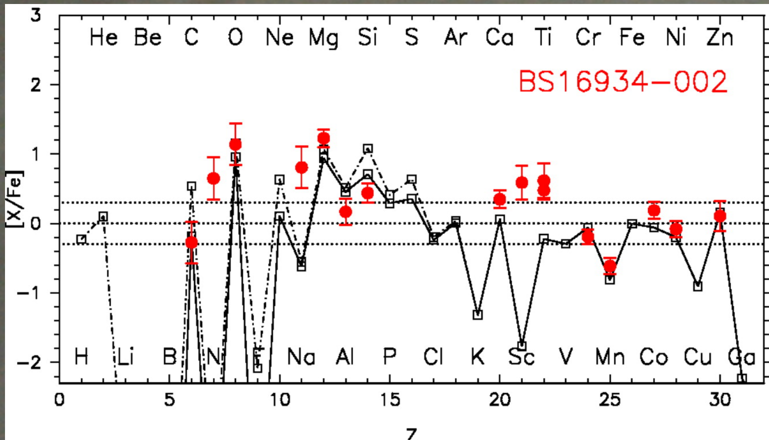




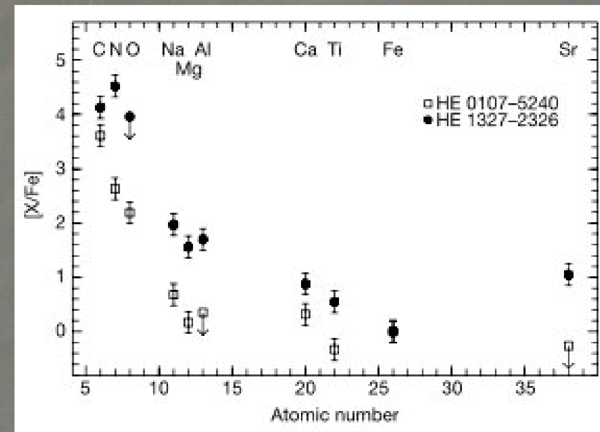
Asplund M, et al. 2009.
Annu. Rev. Astron. Astrophys. 47:481–522



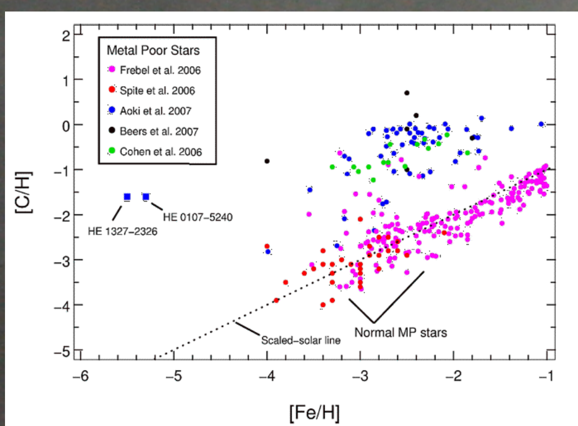
C
N
O
Na
Mg
Al
Si
S
K
Ca
Sc
Ti
V
Cr
Mn
Co
Ni
Cu
Zn
[Fe/H]
Slide stolen from Nikos Prantzos

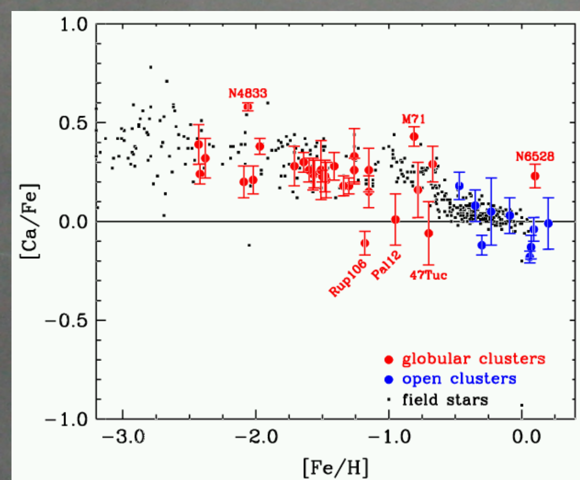


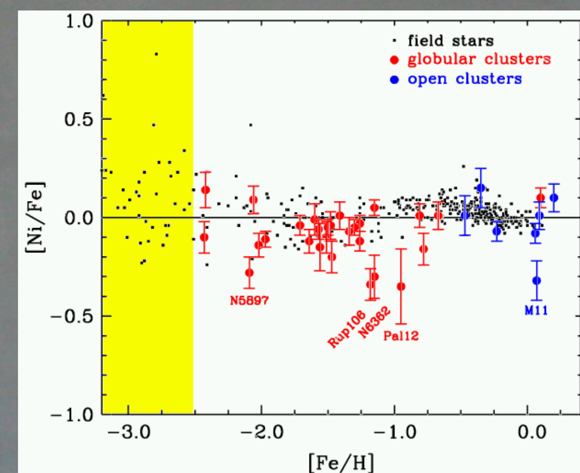
Aoki et al. 2007 ApJ 660,747



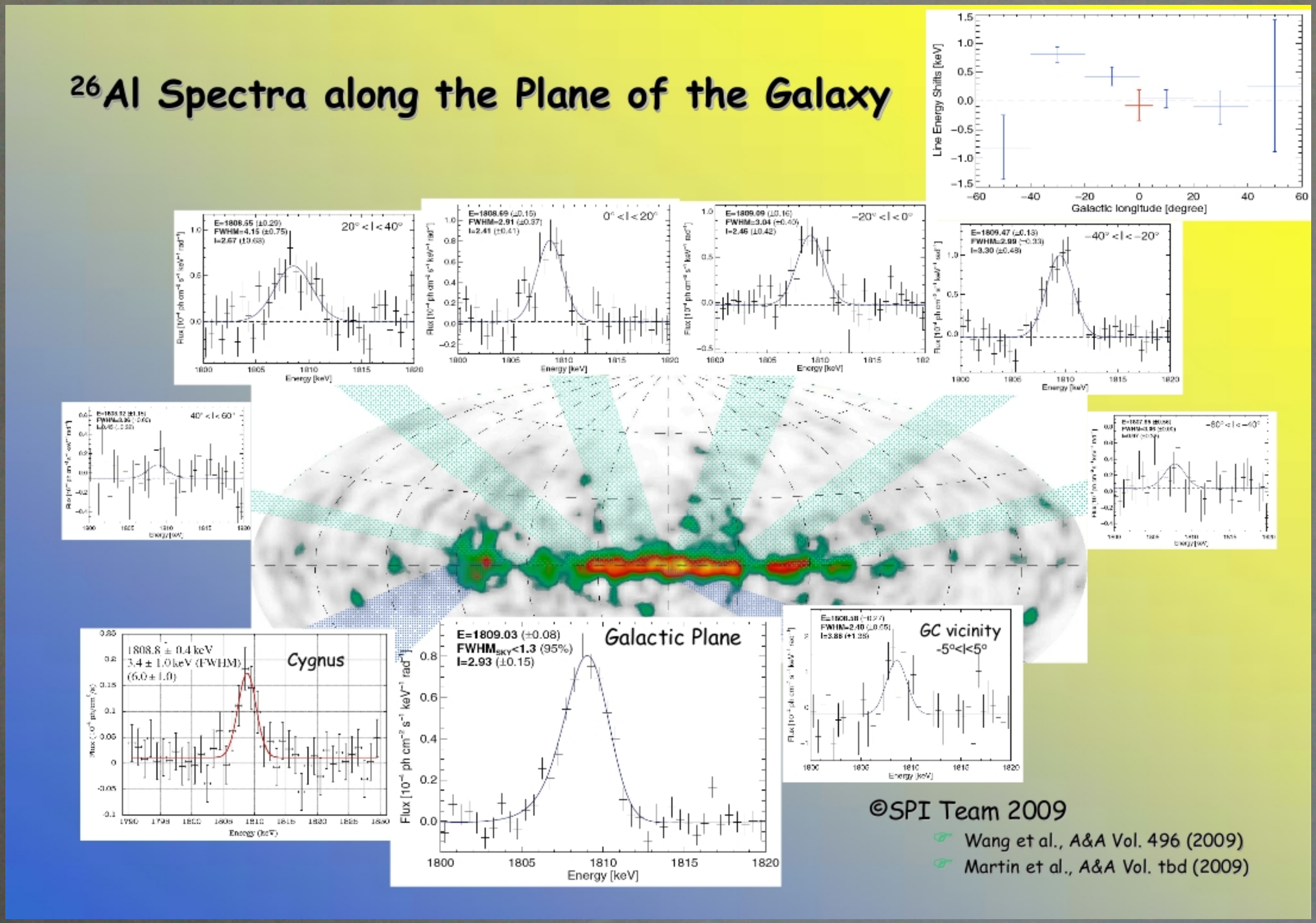
Frebel et al. 2005 Nature 434,871





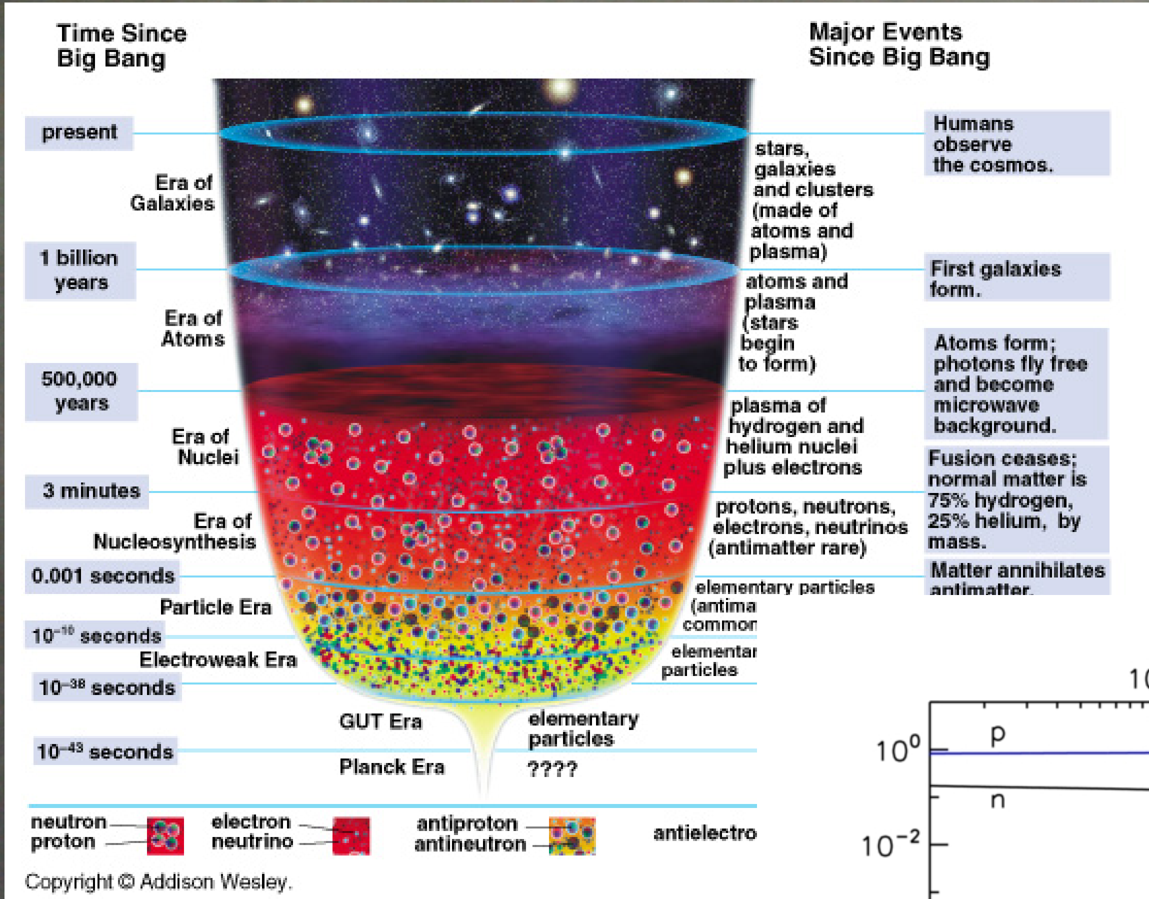


Courtesy: C. Sneden



+

^{60}Fe ^{44}Ti ^{56}Ni


Time Since Big Bang
Major Events Since Big Bang
present
Era of Galaxies
1 billion years
Era of Atoms
500,000 years
Era of Nuclei
3 minutes
Era of Nucleosynthesis
0.001 seconds
Particle Era
10^{-10} seconds
Electroweak Era
10^{-38} seconds
GUT Era
10^{-43} seconds
Planck Era
stars, galaxies and clusters (made of atoms and plasma)
atoms and plasma (stars begin to form)
plasma of hydrogen and helium nuclei plus electrons
protons, neutrons, electrons, neutrinos (antimatter rare)
elementary particles
elementary particles
????
Humans observe the cosmos.
First galaxies form.
Atoms form; photons fly free and become microwave background.
Fusion ceases; normal matter is 75% hydrogen, 25% helium, by mass.
Matter annihilates antimatter.
neutron
proton
electron
neutrino
antiproton
antineutron
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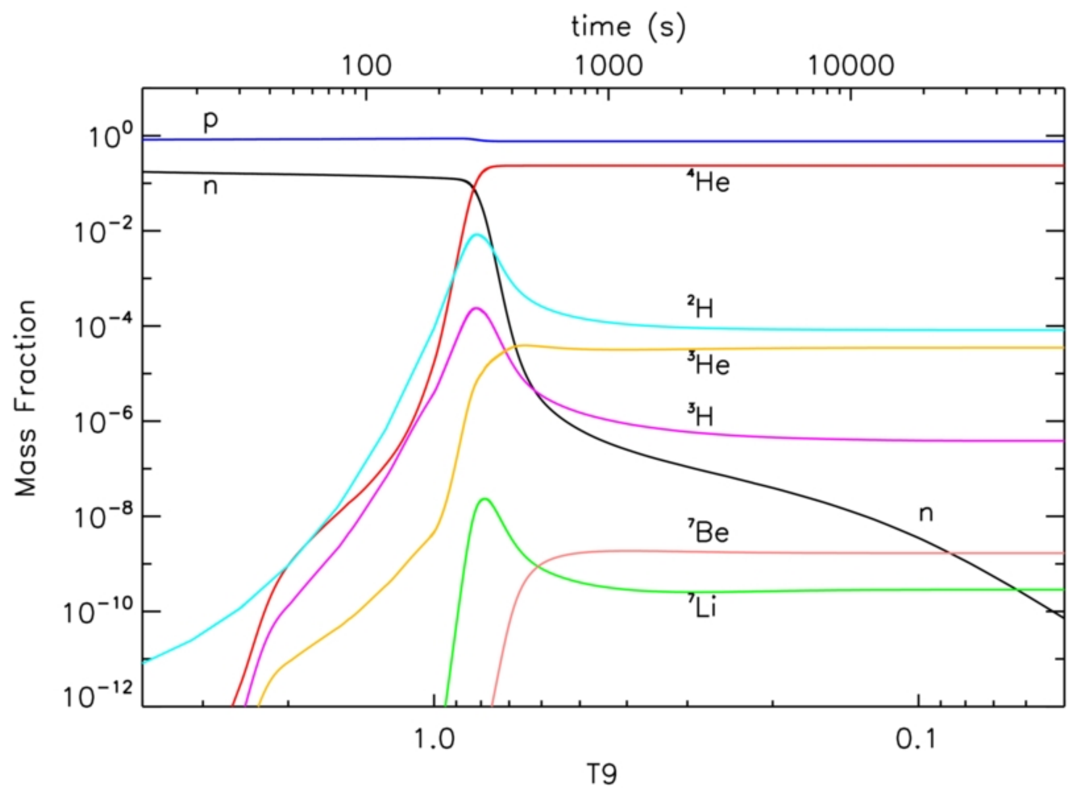
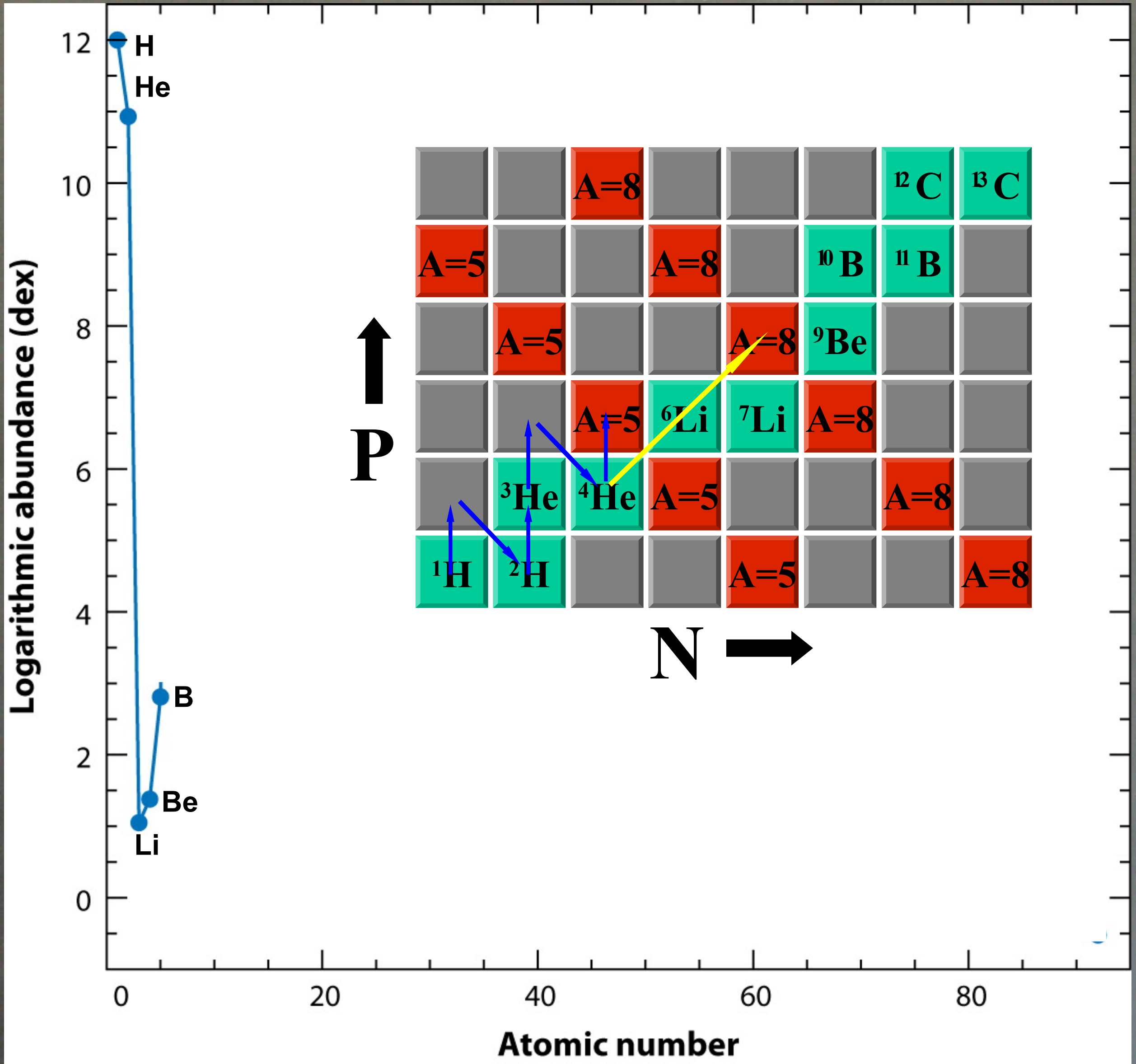

time (s)
100
1000
10000
Mass Fraction
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
p
n
^{4}He
^{2}H
^{3}He
^{3}H
n
^{7}Be
^{7}Li
1.0
0.1
T9



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Evidence

The **Coulomb barrier** prevents an easy fusion between charged particles: only a combination of **high temperatures**, **high densities** and **long timescales** may lead to a substantial amount of fusion.

Even the fusion of the lightest nuclei, protons, requires

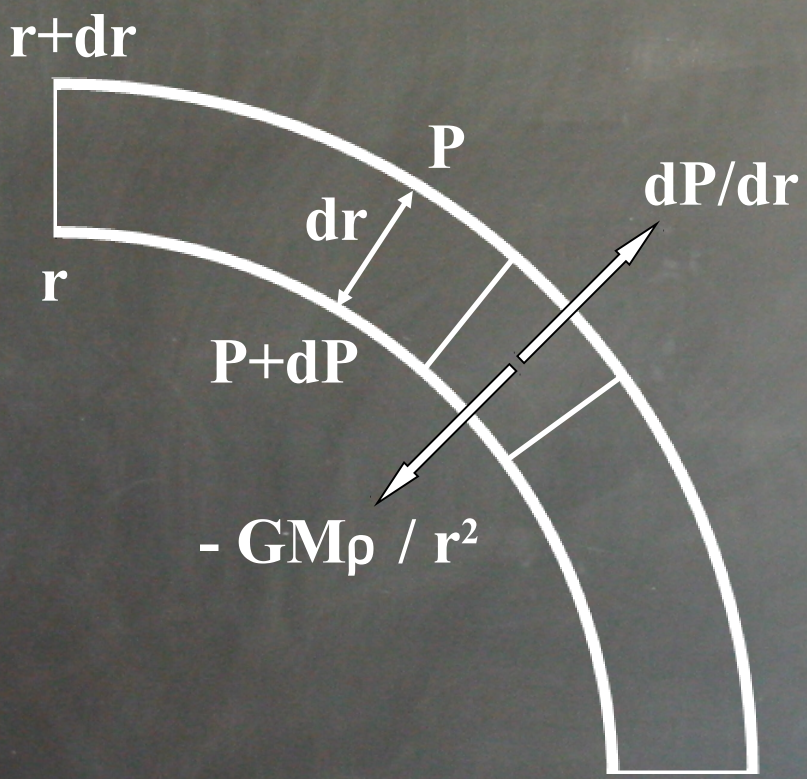
$T >$ several 10^6 K
$\rho >$ several grams / cm^3

to burn a significant amount of nuclei on a timescale shorter than the age of the Universe

These conditions are met only in stars

A star is formed by a gas cloud that contracts under its own gravity and whose luminosity is produced in its interior

In many cases the contraction occurs on "long" timescales because matter naturally settles on a quasi equilibrium configuration in which the various forces acting on each element of matter tend to counterbalance each other:



Hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

Equation of continuity:

Mass conservation:

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{Gm\,\rho}{r^2}$$

Hydrostatic equilibrium

$$\int_0^M \frac{dP}{dr}\frac{r}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm$$

$$-\int_0^M \frac{Gm}{r}\,dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from "infinity" to the present configuration.

$$\int_0^M \frac{dP}{dr}\frac{r}{\rho}\,dm = \int_0^M \frac{dP}{dm}\frac{dm}{dr}\frac{r}{\rho}\,dm = \int_0^M \frac{dP}{dm}4\pi r^2 \rho \frac{r}{\rho}\,dm = \int_0^M \frac{dP}{dm}4\pi r^3\,dm$$

This can be integrated by parts

$$\left(P\,٤\,\pi\, r^{٣}\right)_{٠}^{M} - \int_{٠}^{M} P\,٤\,\pi \frac{dr^{\square}}{dm}\,dm = -\int_{٠}^{M} P\,٤\,\pi\,٣\, r^{٢}\frac{dr}{dm}\,dm = -\int_{٠}^{M} P\frac{٣}{\rho}\,dm$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$ ⬅ **Hydrostatic equilibrium**

$$\int_0^M \frac{dP}{dr}\frac{r}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm$$

$$-\int_0^M 3\frac{P}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from "infinity" to the present configuration.

At this point we need an equation of state, i.e. a relation between pressure and density

Let us firstly consider a perfect gas; in this case we can write:

$$\left.\begin{array}{l} E = \frac{3}{2}NKT \\ P = NKT \end{array}\right\} P = \frac{2}{3}E \Rightarrow \frac{P}{\rho} = \frac{2}{3}u$$

Where u represents the energy per unit mass

Virial Theorem (perfect gas)

$$-\int_0^M 3\frac{2}{3}u\,dm = \Omega \Rightarrow -2\underbrace{\int_0^M u\,dm}_{=E_i} = \Omega \Rightarrow -2E_i = \Omega$$

$= E_i$ <= *total internal energy*

Virial theorem (perfect gas)

$$2E_i + \Omega = 0$$

What does it mean?

$$\Delta E_i = -\frac{1}{2}\Delta\Omega$$

$\Delta\Omega$ is negative! hence a contraction implies necessarily an increase of the internal energy E_i. However only 50% of the energy gained by the gravitational field remains locked in the star, the other 50% must be lost!

The requirement that some energy must be lost in a contraction introduces the idea that the contraction requires some finite timescale to occur, it cannot occur instantaneously. Since energy is basically lost through photons from the surface, this timescale is dictated by the efficiency of the outward photon flux. In other words no additional contraction may occur until the energy losses required by the virial theorem have been effectively lost!

What about the total energy of the system?

$$E_{TOT} = E_i + \Omega \qquad E_{TOT} = -\frac{1}{2}\Omega + \Omega \qquad E_{TOT} = \frac{1}{2}\Omega \qquad \Delta E_{TOT} = \frac{1}{2}\Delta\Omega$$

Once again, Ω is negative! Hence a contraction ($\Delta\Omega<0$) implies a reduction of the total energy.

The system is more bound!

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$ $\Leftarrow$ **Hydrostatic equilibrium**

$$\int_0^M \frac{dP}{dr}\frac{r}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm$$

$$-\int_0^M 3\frac{P}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm = \Omega$$

Gravitational potential energy

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Where *u* represents the energy per unit mass

Virial Theorem (perfect gas)

$$-\int_0^M 3\frac{2}{3}u\,dm = \Omega \;\Rightarrow\; -2\int_0^M u\,dm = \Omega \;\Rightarrow\; -2E_i = \Omega$$

$= E_i$ <= *total internal energy*

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$ ⬅ **Hydrostatic equilibrium**

$$-\int_0^M 3\frac{P}{\rho}dm = -\int_0^M \frac{Gm}{r}dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from “infinity” to the present configuration.

At this point we need an equation of state, i.e. a relation between pressure and density

In general we may write:

$$\frac{P}{\rho} = (\gamma - 1)U$$

Where γ is the ratio of the specific heats at constant pressure C_P and constant volume C_V

$$dQ = dU + P\,dV$$

$$\left(\frac{dQ}{dT}\right)_V = \left(\frac{dU}{dT}\right)_V = C_V \Rightarrow U = C_V T$$

$$\left(\frac{dQ}{dT}\right)_P = C_P = \left(\frac{dU}{dT}\right)_P + P\left(\frac{dV}{dT}\right)_P \qquad C_P = C_V + \frac{P}{\rho T}\left(\frac{d\ln V}{d\ln T}\right)_P$$

1 for a perfect gas

$$\frac{P}{\rho} = (C_P - C_V)T\left(\frac{d\ln T}{d\ln V}\right)_P$$

$$\frac{P}{\rho} = (C_P - C_V)\frac{U}{C_V} = (\gamma - 1)U$$

For a monoatomic gas γ = 5/3 so that we re - obtain

$$\frac{P}{\rho} = \left(\frac{5}{3} - 1\right)U = \frac{2}{3}U$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$ ⟸ **Hydrostatic equilibrium**

$$-\int_0^M 3\frac{P}{\rho}\,dm = -\int_0^M \frac{Gm}{r}\,dm = \Omega$$

Gravitational potential energy

Ω may be regarded as the total amount of gravitational energy liberated in the contraction from “infinity” to the present configuration.

Generalized Virial Theorem

$$-\int_0^M 3(\gamma-1)U\,dm = \Omega \qquad 3(\gamma-1)E_i + \Omega = 0$$

$$\gamma = \frac{4}{3}$$ **is very “special” because in this case**

$$E_i + \Omega = 0 \qquad \Delta E_i = -\Delta\Omega$$

All the energy gained by the gravitational field is stored in the star (as internal energy) and no energy is lost outward.

$$\Delta E_{TOT} = \Delta E_i + \Delta\Omega = -\Delta\Omega + \Delta\Omega = 0$$

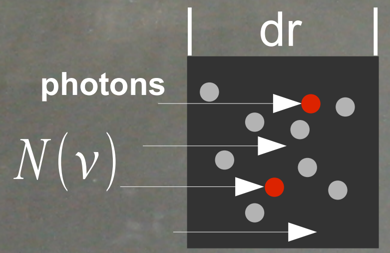
For γ=4/3 a contraction does not increase the binding energy but leaves E_{TOT} constant!

Since the contraction does not require the ejection of any energy no “delay” is necessary for a further contraction to occur. This is an unstable situation that leads to the collapse of the structure

The second basic equation necessary to describe a stellar structure is the one that controls the energy transport through the star.

Let us firstly assume that the energy is transported by radiation only:

The momentum (*dq*) transferred by a flux N of photons of frequency ν per unit time is given by:



λ mean free path

$$\lambda = \frac{1}{\kappa \rho}$$

Opacity coefficient

Mean number of interactions

$$dq = N(\nu)\frac{h\nu}{c}\frac{dr}{\lambda}dS\,dt = \frac{L\kappa\rho}{4\pi R^2 c}dr\,dS\,dt$$

Momentum per photon

But, the momentum transferred may be also expressed as the variation of the radiation pressure:

$$dq = -dP_r\,dS\,dt = -\frac{1}{3}a\,dT^4\,dS\,dt = -\frac{4}{3}aT^3\,dT\,dS\,dt$$

By equating the two:

$$\frac{4}{3}aT^3\frac{dT}{dr} = -\frac{L\kappa\rho}{4\pi R^2 c} \Longrightarrow \frac{dT}{dr} = -\frac{3}{16\pi a c}\frac{\kappa\rho L}{R^2 T^3}$$

2nd basic equation

Associated continuity equation:

$$\frac{dL}{dM} = \epsilon = \epsilon_{nuc} + \epsilon_{grav} - \epsilon_{\nu}$$

SUMMARIZING

The set of equations that describe the structure of a star is given by:

$$\frac{dP}{dr} = -\frac{G m \rho}{r^2}$$ Hydrostatic equilibrium

$$\frac{dM}{dr} = 4\pi r^2 \rho$$ Mass conservation

$$\frac{dT}{dr} = -\frac{3}{16 a c \pi} \frac{\kappa \rho L}{r^2 T^3}$$ Energy transport (radiative case)

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$ Energy conservation

+

Equation Of State, i.e. P (ρ, T, c.c.)

Opacity coefficient, i.e. κ (ρ, T, c.c.)

Energy generation coefficient, i.e. ε (ρ, T, c.c.)

The solution of this system of equations is very difficult and requires COMPUTERS!

but...

...we can try to be clever!

Perfect gas

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \Rightarrow \frac{P}{R} \propto \frac{M\rho}{R^2} \Rightarrow P \propto \frac{M\rho}{R}$$

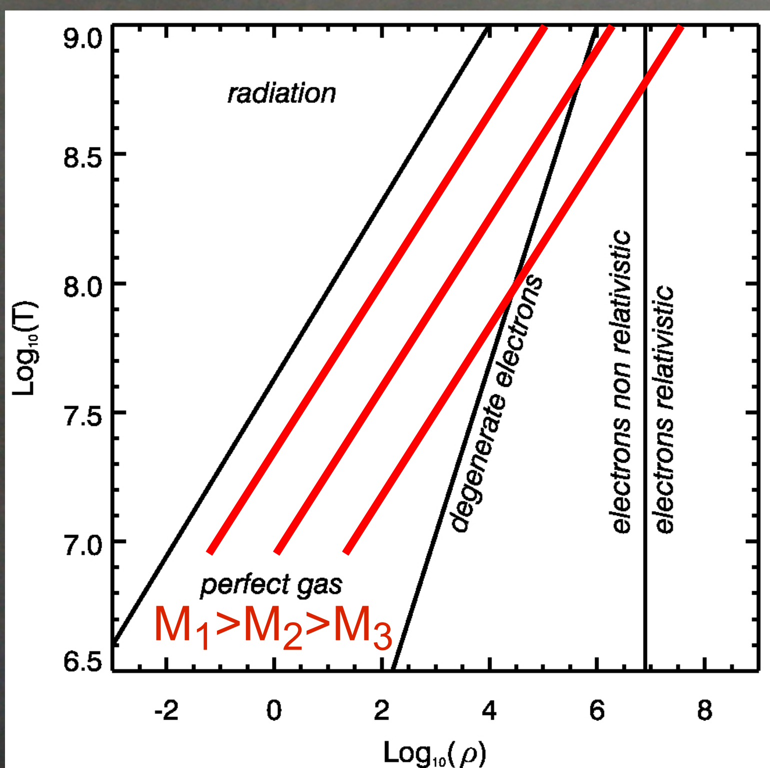
$$\frac{dM}{dr} = 4\pi r^2 \rho \Rightarrow \frac{M}{R} \propto R^2 \rho \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}}$$

$$P \propto M^{\frac{2}{3}} \rho^{\frac{4}{3}}$$

$$P \propto \rho T$$

$$\frac{T^3}{\rho} \propto M^2$$

$$\log(T) = K(M) + \frac{1}{3}\log(\rho)$$



Interesting!

Just the hydrostatic equilibrium + perfect gas imply that the centre of a star must evolve along a straight line in the Log(T_c)-Log(ρ_c) plane.

What else?

The constant k scales inversely with the mass, so that the density increases as the mass decreases (for each fixed T)

We found that stars naturally separate in two basic groups: stars less massive than a critical value enter the region of electron degeneracy while the more massive ones don't!

Perfect gas

$$\frac{dP}{dr}=-\frac{Gm\rho}{r^2} \Rightarrow \frac{P}{R}\propto\frac{M\rho}{R^2} \Rightarrow P\propto\frac{M\rho}{R}$$

$$\frac{dM}{dr}=4\pi r^2\rho \Rightarrow \frac{M}{R}\propto R^2\rho \Rightarrow R\propto\left(\frac{M}{\rho}\right)^{\frac{1}{3}}$$

$$P\propto M^{\frac{2}{3}}\rho^{\frac{4}{3}}$$

$$P\propto\rho T$$

$$TR\propto M$$

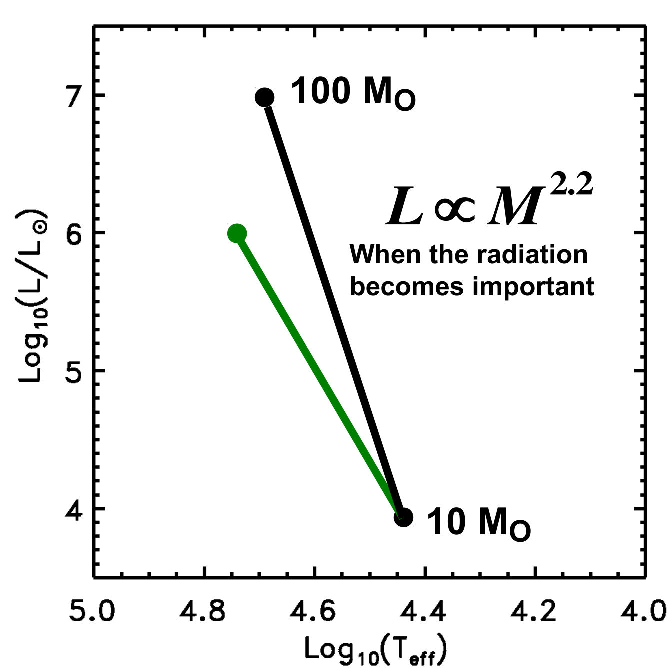
$$\frac{dT}{dr}=-\frac{3}{16\,a\,c\,\pi}\frac{k\rho L}{r^2T^3} \Rightarrow \frac{T}{R}\propto\frac{\rho L}{R^2T^3} \Rightarrow T^4R^4\propto ML \Rightarrow L\propto M^3$$

What about the surface temperature of the star?

If we assume a black body: $L=4\pi R^2\sigma T_{eff}^4$

and also that the central temperature is roughly independent on the mass : $R\propto M$

$$M^3\propto R^2T_{eff}^4 \Rightarrow T_{eff}^4\propto M \Rightarrow T_{eff}\propto M^{\frac{1}{4}}$$



Perfect gas

$$P \propto \rho T$$

$$TR \propto M$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \Rightarrow \frac{P}{R} \propto \frac{M\rho}{R^2} \Rightarrow P \propto \frac{M\rho}{R}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \Rightarrow \frac{M}{R} \propto R^2 \rho \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}}$$

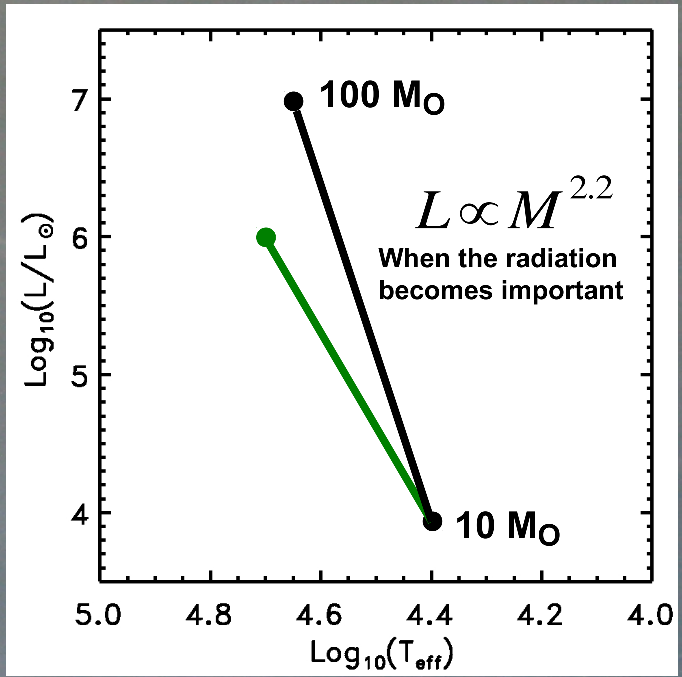
$$\Bigg\} \; P \propto M^{\frac{2}{3}} \rho^{\frac{4}{3}}$$

$$\frac{dT}{dr} = -\frac{3}{16\,a\,c\,\pi} \frac{k\rho L}{r^2 T^3} \Rightarrow \frac{T}{R} \propto \frac{\rho L}{R^2 T^3} \Rightarrow T^4 R^4 \propto ML \Rightarrow L \propto M^3$$

What about the lifetime of the stars?

$$\tau \propto \frac{E}{L} \qquad \tau \propto \frac{qM}{L} \qquad \tau \propto \frac{qM}{M^3} \approx \frac{1}{M^2}$$

When radiation contributes significantly to the EOS $\Rightarrow$ $\tau \propto \frac{qM}{M^{2.2}} \approx \frac{1}{M^{1.2}}$



We learned a lot of things up to now (without really solving any equation!)

If the EOS is dominated by a perfect gas:

hydrostatic equlibrium is “stable” because γ (C_p/C_v) > 4/3

the evolution of the core follows a straight line in the $Log(T_c)$-$Log(\rho_c)$ plane

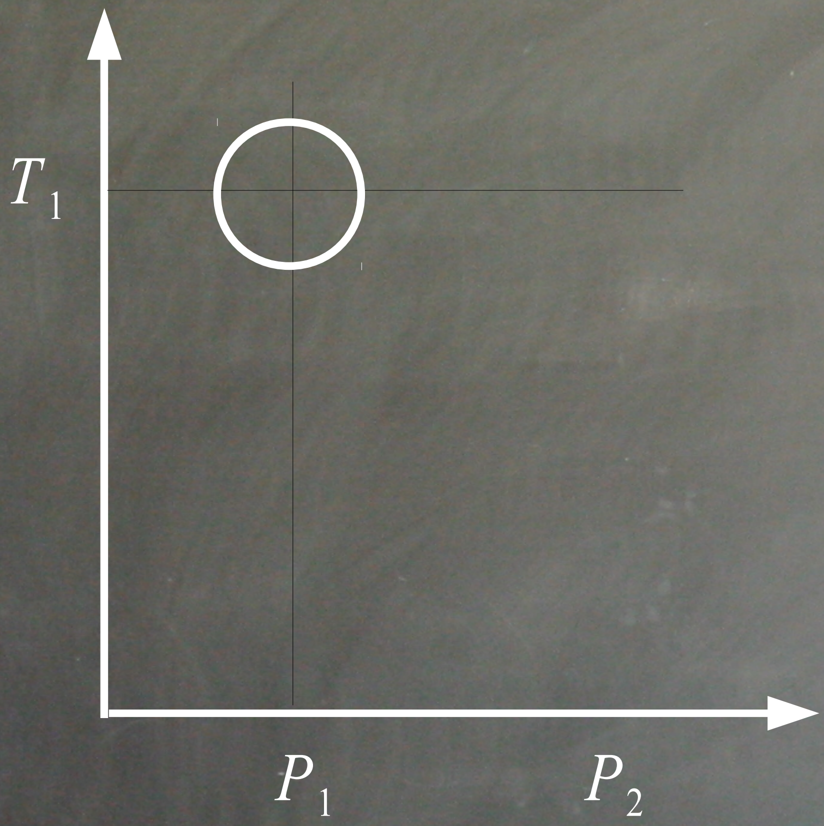
Star less massive than a critical value enter the region where degenerate electrons count

Star more massive than a critical value do not enter the region where degenerate electrons count (at least until the central temperature does not exceed a few billions of K)

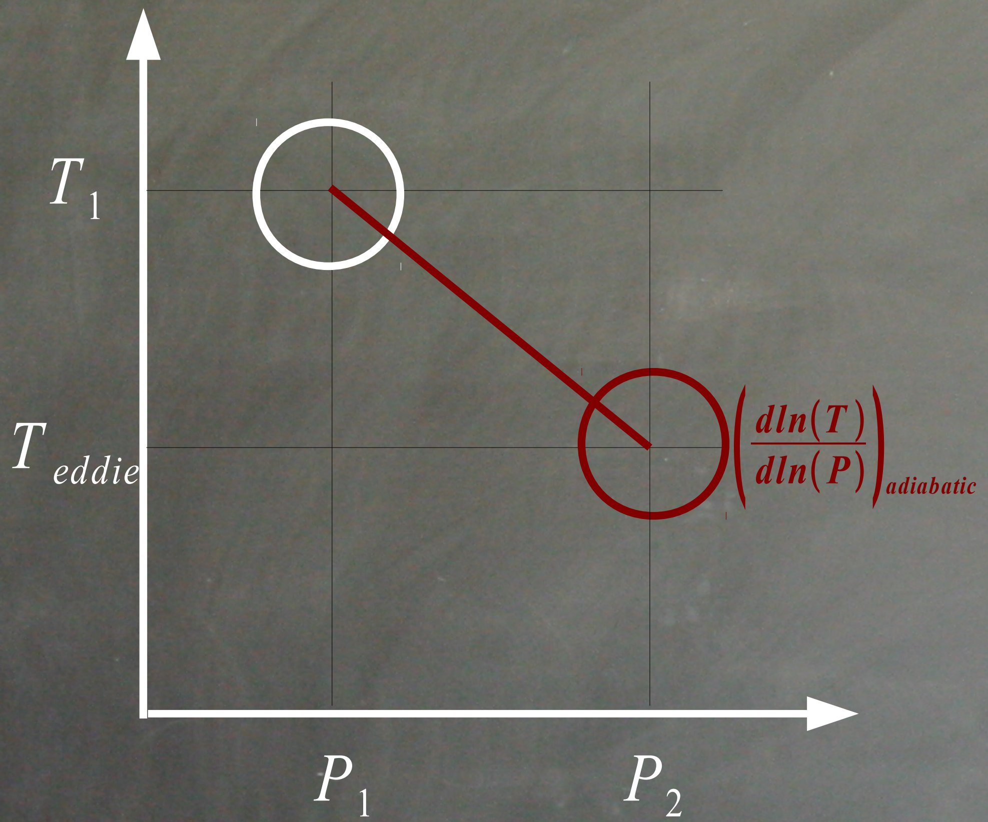
The energy losses from the surface (L) scale as M^3

The lifetime of a star (t) scales as M^{-2}

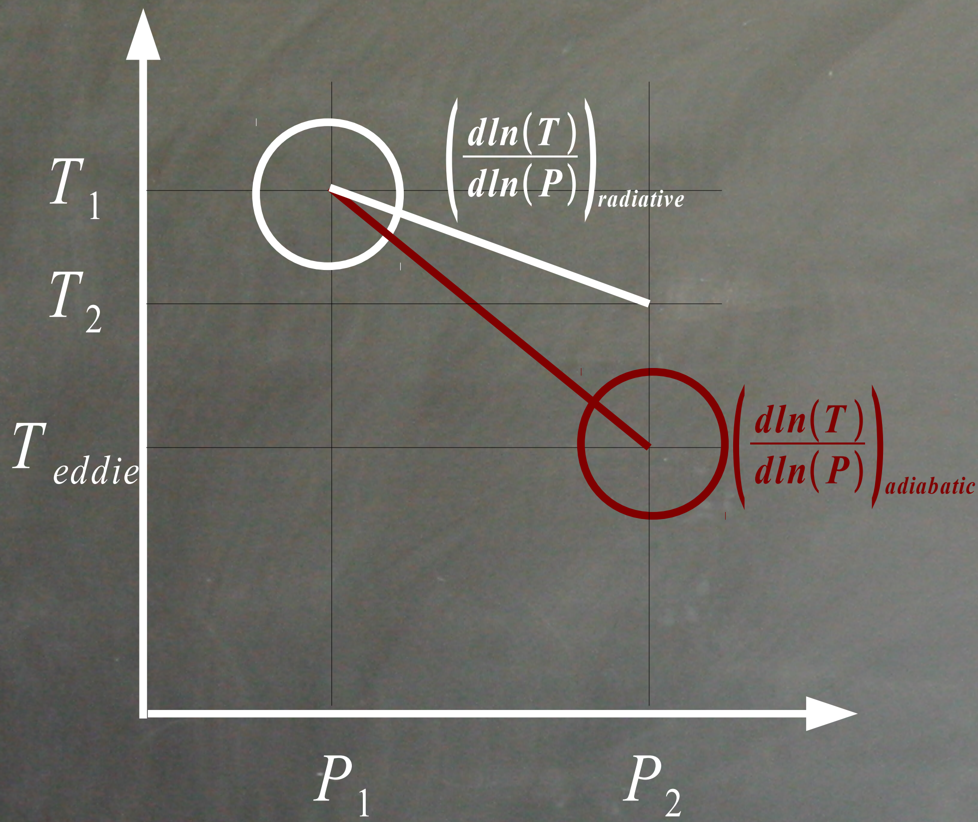
Unfortunately this is not enough ... it's time to introduce CONVECTION



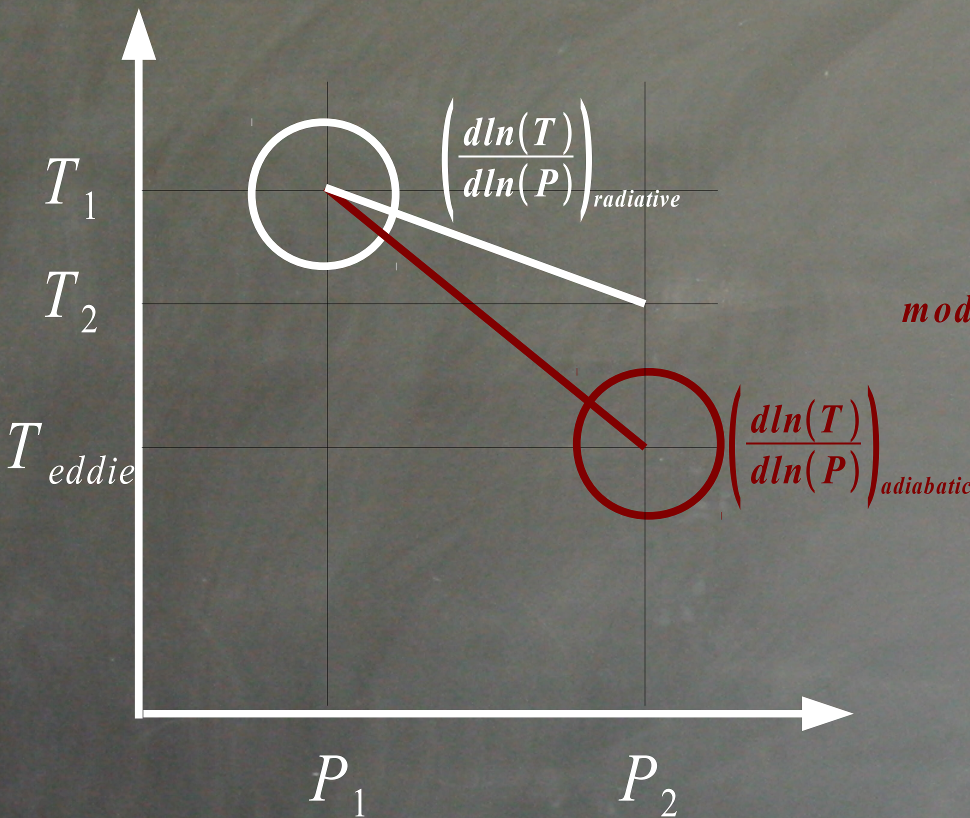
T_1
P_1
P_2



T_1
T_{eddie}
$\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic}$
P_1
P_2



T_1
T_2
T_{eddie}
P_1
P_2
$\left(\frac{dln(T)}{dln(P)}\right)_{radiative}$
$\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic}$



$$mod\left(\frac{d\ln(T)}{d\ln(P)}\right)_{adiabatic} > mod\left(\frac{d\ln(T)}{d\ln(P)}\right)_{radiative}$$

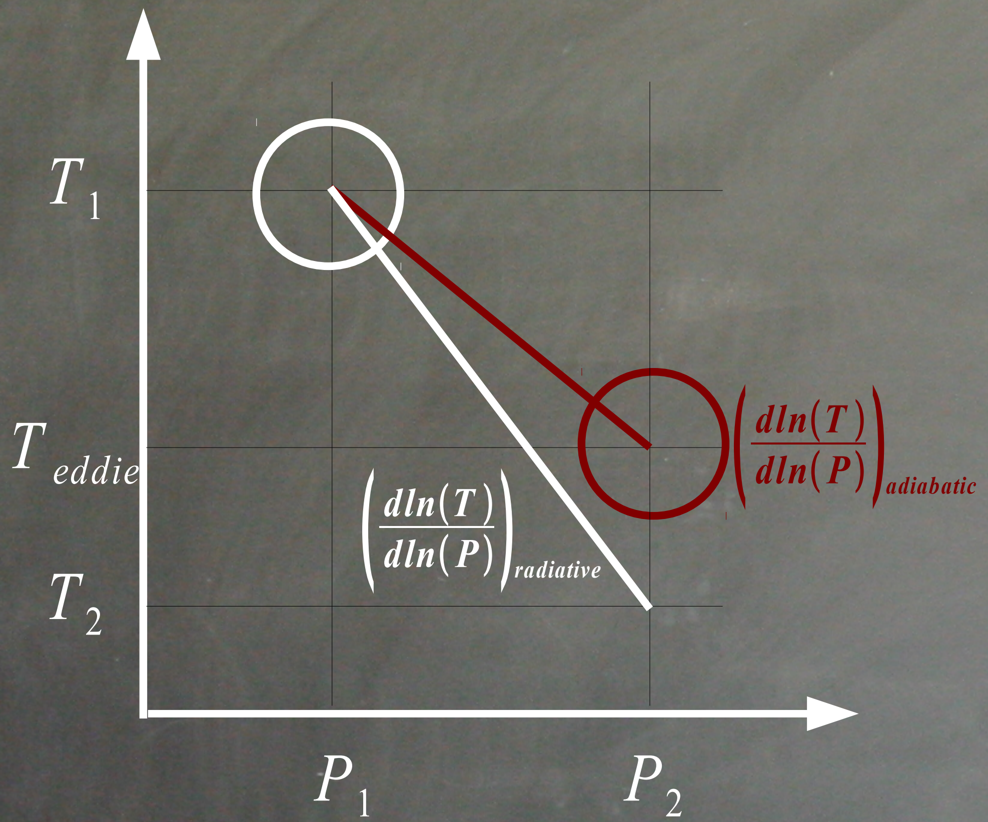
$$P \propto \rho T = const$$

$$T_{eddie} < T_2$$

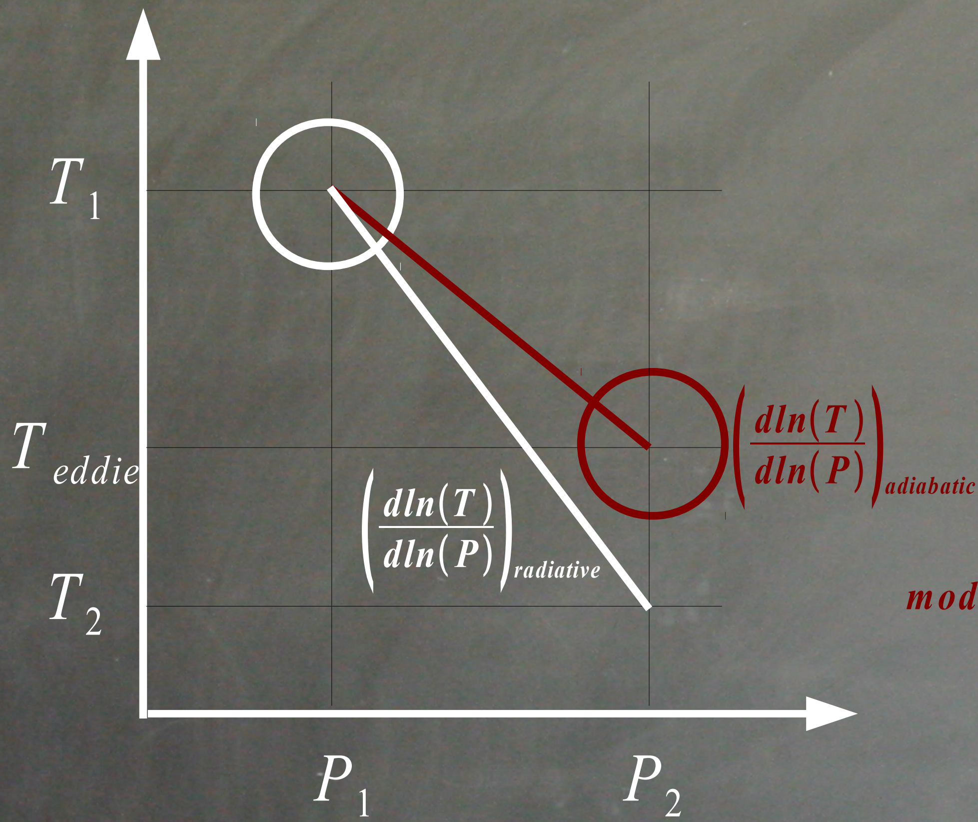
$$\rho_{eddie} > \rho_2$$

The buoyancy force $f \approx -\frac{g}{\rho}\frac{\delta \rho}{\delta r}\Delta r$

is negative and pushes back the eddie



T_1
T_{eddie}
T_2
P_1
P_2
$\left(\frac{dln(T)}{dln(P)}\right)_{radiative}$
$\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic}$



$$mod\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic} < mod\left(\frac{dln(T)}{dln(P)}\right)_{radiative}$$

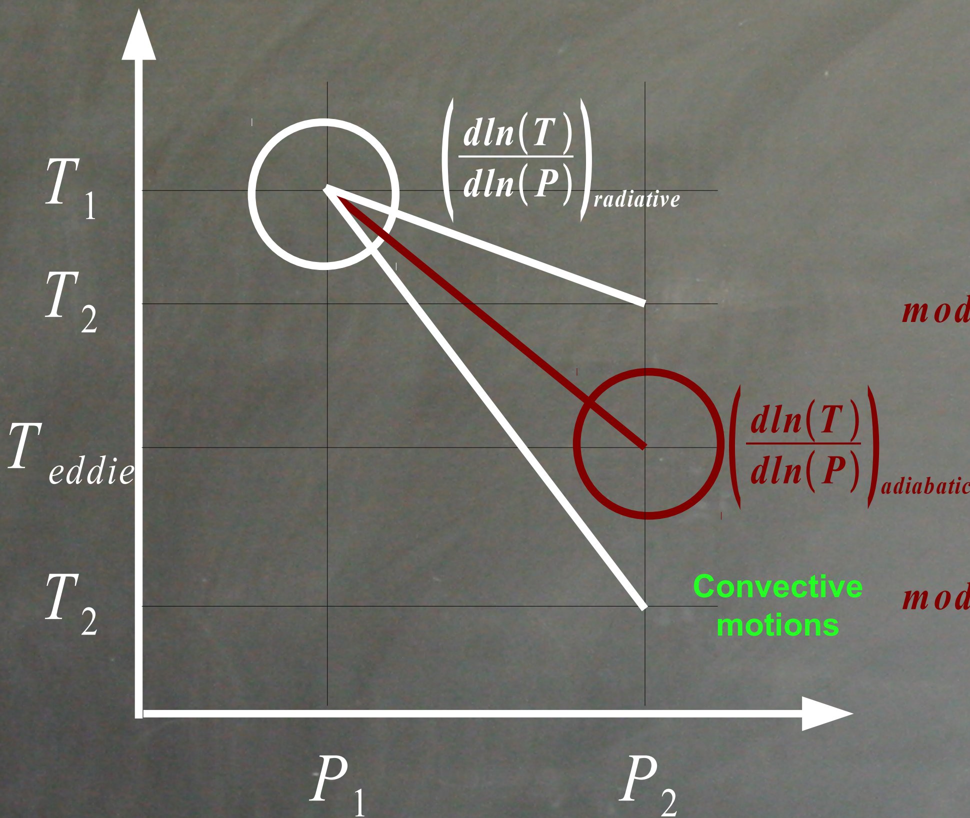
$$T_{eddie} > T_2$$

$$\rho_{eddie} < \rho_2$$

The eddie is accelerated outward.
Large scale motions activate.

The buoyancy force $f \approx -\frac{g}{\rho}\frac{\delta\rho}{\delta r}\Delta r$ **is now positive**

Schwarzschild criterion!



$$P \propto \rho T = const$$

$$mod\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic} > mod\left(\frac{dln(T)}{dln(P)}\right)_{radiative} \qquad T_{eddie} < T_2 \qquad \rho_{eddie} > \rho_2$$

$$mod\left(\frac{dln(T)}{dln(P)}\right)_{adiabatic} < mod\left(\frac{dln(T)}{dln(P)}\right)_{radiative} \qquad T_{eddie} > T_2 \qquad \rho_{eddie} < \rho_2$$

Both the temperature gradient and the mass extension of the convective regions are very difficult to compute properly and still constitute one of the major uncertainties in the stellar modelling.

Which are the basic consequences of the growth of convective motions?

Matter is mixed.

1st side effect: new fuel pulled inward – products of burning pushed outward

2rd side effect: change of the mean molecular weight in the whole convective region

The temperature gradient can't become steeper than the adiabatic one in most of the interior of a star; only in the outer region it can raise towards the radiative one because of the inefficiency of the eddies in carrying the energy.

At this point we are ready to follow the evolution of a star, but...

...first a “stupid” question...

Why should a star “evolve”?
(i.e. change its structure as time goes by)

because...

...stars lose energy (e.g. from the surface: the Luminosity)

that must be replaced in order to mantain the hydrostatic equilibrium!

Energy may be gained by either:

contraction (energy is extracted from the gravitational field)

Side effect => the interior heats (Virial theorem)

and / or

Nuclear reactions (energy is extracted from the fusion of nuclei)

Side effect => mean molecular weight increases (P decreases)

H-burning

He-burning

C-burning

Ne-burning

O-burning

Si-burning

Core collapse

Bounce at nuclear densities
Formation of the shock wave

Explosive nucleosynthesis
yields

T

Hydrostatic
evolution

Hydrodynamic
evolution

Nuclear reaction rates

Let us define the cross section σ_{ij} of the nuclear reaction between the particles i and j, that has the exit channel k + l, as:

$$\sigma_{ij} = \frac{number\ of\ reactions\ (particles)^{-1} \cdot s^{-1}}{flux\ of\ particles}\quad cm^2$$

The total number of reactions between the subset of particles i and j that have relative velocity v is given by:

$$R_{ij}(v) = n_i \cdot v \cdot n_j \sigma_{ij}(v) \quad numbers^{-1} \cdot cm^{-3}$$

Where n_i and n_j represent the number densities of the two nuclear species having relative velocity v.

Since the product n_i n_j represents the number of pairs, the product v σ_{ij} may be seen as the probability (per single pair) that the given process i(j,k)l occurs.

$$R_{ij}(v) = n_i \cdot n_j \cdot v \cdot \sigma_{ij}(v)$$

If both the i and j components of the gas behave as an “ideal” gas, their velocity distribution is Maxwellian. As a consequence also their relative velocity distribution is Maxwellian :

$$n_i \cdot n_j = N_i \cdot N_j \left(\frac{2}{\pi}\right)^{1/2} \frac{\mu^{3/2}}{(KT)^{3/2}} \cdot v^2 \cdot e^{-\frac{\mu v^2}{2KT}}$$

Where N_i and N_j represent the total number densities of the two species, and m the reduced mass:

$$\mu = \frac{(A_i A_j)}{(A_i + A_j)}$$

The rate R_{ij} may be therefore be written as:

$$R_{ij}(v) = N_i \cdot N_j \left(\frac{2}{\pi}\right)^{1/2} \frac{\mu^{3/2}}{(KT)^{3/2}} \cdot v^2 \cdot e^{-\frac{\mu v^2}{2KT}}\ v\,\sigma_{ij}(v)$$

Nuclear reaction rates

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By integrating over the velocity distribution we get:

$$R_{ij} = N_i \cdot N_j \left(\frac{2}{\pi}\right)^{1/2} \frac{\mu^{3/2}}{(KT)^{3/2}} \cdot \int_0^\infty v^2 \cdot e^{-\frac{\mu v^2}{2KT}} v\,\sigma_{ij}(v)\; dv$$

By converting the velocity in energy, R_{ij} becomes:

$$R_{ij} = N_i \cdot N_j \left(\frac{8}{\pi\,\mu}\right)^{1/2} \frac{1}{(KT)^{3/2}} \cdot \int_0^\infty E \cdot e^{-\frac{E}{KT}} \sigma_{ij}(E)\, dE$$

Nuclear reaction rates

By converting the velocity in energy, R_{ij} becomes:

$$R_{ij} = N_i \cdot N_j \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(KT)^{3/2}} \cdot \int_0^\infty E \cdot e^{-\frac{E}{KT}} \sigma_{ij}(E)\, dE$$

The cross section $\sigma_{ij}(E)$ may be written as the product of two parts:

$$\sigma_{ij}(E) = P_C(E) \cdot S(E)$$

$P_C(E)$ represents the probability that the two particles arrive at nuclear distance (10^{-15} cm). This problem has been address by George Gamow and hence it is usually know as the Gamow factor:

$$P_C(E) = \frac{1}{E} \cdot e^{-\frac{4\pi^2 Z_i Z_j e^2}{h\sqrt{E}}}$$

Tunnel effect

Coulomb repulsion

S(E) represents the probability that the compound nucleus decays in the desired channel k+l. It is called the Astrophysical factor because it contains the nuclear properties of the process.

$$R_{ij}(T) = N_i \cdot N_j \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(KT)^{3/2}} \cdot \int_0^\infty E \cdot e^{-\frac{E}{KT}} \frac{1}{E} \cdot e^{-\frac{4\pi^2 Z_i Z_j e^2}{h\sqrt{E}}} S(E)\, dE$$

$$R_{ij}(T) = N_i \cdot N_j \cdot A(T) \cdot \int_0^\infty e^{-B(T)E} \cdot e^{-\frac{C_{ij}}{\sqrt{E}}} S(E)\, dE$$

B(T) and C_{ij} always positive!

Nuclear reaction rates

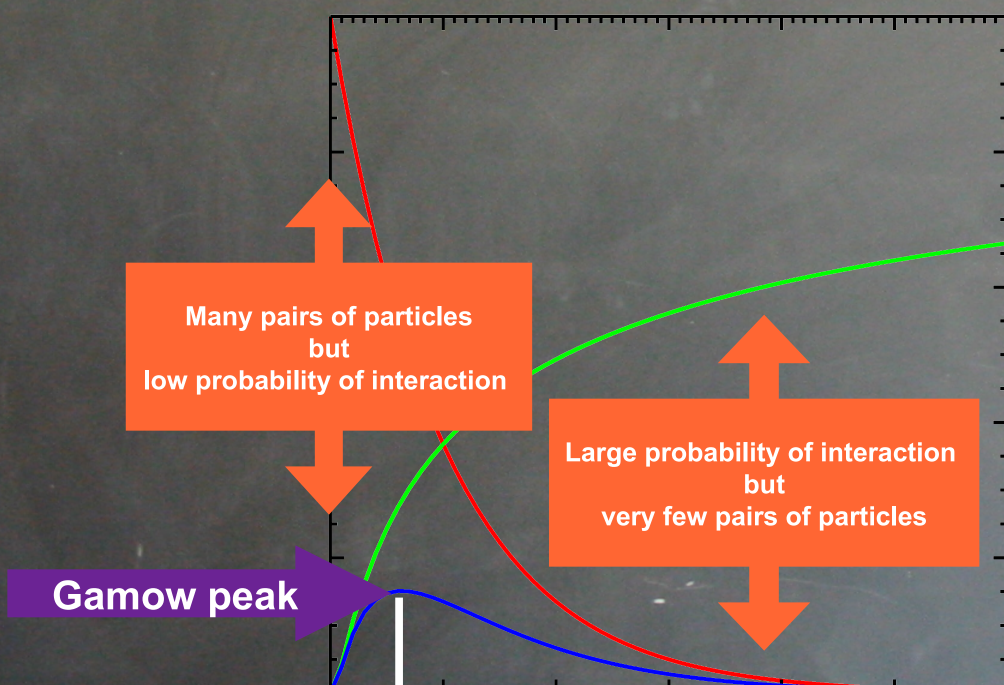
Let us look at the integrand a little bit closer

$$R_{ij}(T) = N_i \cdot N_j \cdot A(T) \cdot \int_0^\infty e^{-B(T)E} \cdot e^{-\frac{C_{ij}}{\sqrt{E}}} S(E)\, dE$$

B(T) and C_{ij} always positive!

This term goes to zero as the energy goes to infinity

This term goes to zero as the energy goes to zero



E_{Gamow} must not be confused with KT

The astrophysical factor must be known only in a restricted energy range around the Gamow peak

There is one Gamow peak for each temperature T.
Remember that the Energy of the Gamow peak
is NOT
the energy related to the temperature T (i.e. KT)

The work of the Nuclear Astrophysicists consists in the experimental measure of the Astrophysical factors S(E) for as many processes as possible

Nuclear reaction rates

If we define, for simplicity:

$$\langle \sigma v \rangle_{ij}(T) = \int_0^\infty e^{-B(T)E} \cdot e^{-\frac{C_{ij}}{\sqrt{E}}} S(E)\,dE$$

The rate of a given process i(j,k)l, i.e. the number of interactions per unit time at temperature T is:

$$R_{ij}(T) = N_i \cdot N_j \cdot \langle \sigma v \rangle_{ij}(T)$$

The total number of particles i per unit volume may be easily derived by means of the density ρ and its mass fraction abundance x_i:

$$N_i = x_i \cdot \rho \cdot \frac{N_A}{A_i}$$

Abundance in gr of the reactant i per unit volume

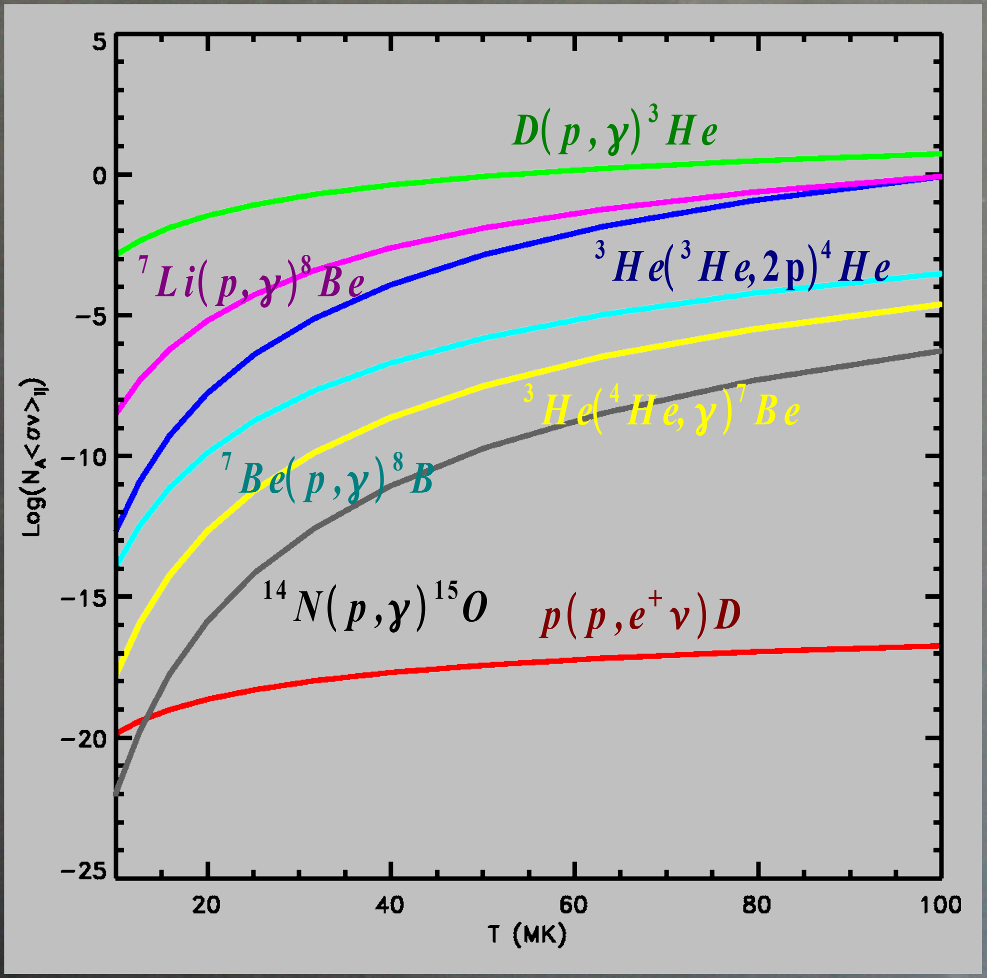
Number of nuclei i per gr

If we define y the number of particles i corresponding to the mass fraction: $y_i = \frac{x_i}{A_i}$

The rate R becomes: $R_{ij}(T) = y_i \cdot y_j \cdot \rho^2 \cdot N_A^2 \langle \sigma v \rangle_{ij}(T)$ # reactions cm $^{-3}$ s $^{-1}$

Nuclear reaction rates

Nuclear Astrophysicists usually provide tables of: $N_A < \sigma v >_{ij}(T)$



Remember that the efficiency, the RATE, of a nuclear process is NOT determined by just the nuclear cross section, but by the full formula:

$$R_{ij}(T) = y_i \cdot y_j \cdot \rho^2 \cdot N_A^2 < \sigma v >_{ij}(T)$$

i.e. the abundances of the reactants i and j are fundamental too!

Critical masses: